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鑑三

G-inflation and its nonGaussianity

Galileon

Jun'ichi Yokoyama

Tsutomu Kobayashi, Masahide Yamaguchi, & JY
"G-inflation: Inflation driven by the Galileon Field"
1008.0603[hep-th] Phys. Rev. Lett. 135(2010)230302.

K. Kamada, T. Kobayashi, M. Yamaguchi, & JY
"Higgs G-inflation" 1012.4238[hep-th], Phys.Rev.D83(2011)083515

T. Kobayashi, M. Yamaguchi, & JY
"Primordial non-Gaussianity from G-inflation"
1103.1740[hep-th], Phys. Rev. D in press.

The Galileon

Nicolis, Rattazzi, & Trincherini 2009

Higher derivative theory with a Galilean shift symmetry $\partial_\mu \phi \to \partial_\mu \phi + \mathrm{const}_\mu$ in the flat space limit.

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = (\nabla \phi)^2$$

$$\mathcal{L}_3 = (\nabla \phi)^2 \Box \phi$$

motivated by brane bending mode in the DGP model at the decoupling limit.

$$\mathcal{L}_4 = (\nabla \phi)^2 \left[2(\Box \phi)^2 - 2(\nabla_\mu \nabla_\nu \phi)^2 - \frac{R}{2}(\nabla \phi)^2 \right]$$

$$\mathcal{L}_5 = (\nabla \phi)^2 \left[(\Box \phi)^3 + \ldots \right]$$

Field equation contains derivatives up to second-order at most.

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nonreleativistic limit of 4D probe brane action in 5D theory (Rham & Tolley 2010)

$$\mathcal{L}_4 = (\nabla\phi)^2 \left[2(\Box\phi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2 - \frac{R}{2}(\nabla\phi)^2 \right]$$

$$\mathcal{L}_5 = (\nabla\phi)^2 \left[(\Box\phi)^3 + \dots \right]$$

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The Galileon

Nicolis, Rattazzi, & Trincherini 2009

Higher derivative theory with a Galilean shift symmetry $\partial_\mu \phi \to \partial_\mu \phi + \mathrm{const}_\mu$ in the flat space limit.

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = \left(\nabla \phi\right)^2$$

$$\mathcal{L}_3 = \left(\nabla \phi\right)^2 \Box \phi$$

quantum corrections to the kinetic term $\phi \Box \phi$

$$\mathcal{L}_4 = \left(\nabla \phi\right)^2 \left[2\left(\Box \phi\right)^2 - 2\left(\nabla_\mu \nabla_\nu \phi\right)^2 - \frac{R}{2}\left(\nabla \phi\right)^2 \right]$$

$$\mathcal{L}_5 = \left(\nabla \phi\right)^2 \left[\left(\Box \phi\right)^3 + \dots \right]$$

Field equation contains derivatives up to second-order at most.

We consider a theory with lowest-order extension in $\Box \phi$.

Our Model

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi, \quad X \equiv -\frac{1}{2}(\partial\phi)^2$$

- Action

$$S = \int \sqrt{-g}\, d^4x \left(\frac{M_{Pl}^2}{2} R + \mathcal{L}_\phi \right)$$

The resultant Einstein and field equations contain derivatives up to second order at most.

- Two extreme cases of the model
  - Kinetically driven G-inflation

$$\mathcal{L}_\phi = K(\cancel{\phi}, X) - G(\cancel{\phi}, X)\Box\phi,$$ No scalar field potential

  - Potential driven G-inflation including Higgs G-inflation

$$\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi,$$

$$K(\phi, X) = X - V(\phi)$$

Background

- Field equations in the homogeneous and isotropic background $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$

$$T_{\mu\nu} = \left(K_X - G_X\right)\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\left(K + \nabla_\lambda G\nabla^\lambda\phi\right) - 2\nabla_{(\nu}G\nabla_{\nu)}\phi$$

$$T^\mu_\nu = \mathrm{diag}\left(-\rho, p, p, p\right) \quad \rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X$$

$$p = K - 2\left(G_\phi + G_X\ddot{\phi}\right)X$$

$$3M_{Pl}^2 H^2 = \rho, \quad M_{Pl}^2\left(3H^2 + 2\dot{H}\right) = -p$$

- Scalar field equation

$$K_X\left(\ddot{\phi} + 3H\dot{\phi}\right) + 2K_{XX}X\ddot{\phi} + 2K_{X\phi}X - K_\phi - 2\left(G_\phi - G_{X\phi}X\right)$$
$$+6G_X\left[\left(HX\right)^{\cdot} + 3H^2X\right] - 4G_{X\phi}X\ddot{\phi} - 2G_{\phi\phi}X + 6HG_{XX}X\dot{X} = 0$$

indeed depends on up to 2nd order derivative only.

Only two of the above three equations suffice.

Kinetically driven G-inflation: Background

- $\mathcal{L}_\phi = K(\phi, X) - G(\phi, X)\Box\phi,$

consider a simple model with a shift symmetry $\phi \to \phi + \text{const.},$

$$K(\phi, X) \equiv K(X), \quad G(\phi, X) \equiv gX \equiv X/M^3.$$

and seek for a solution with $H = \text{const.}$ and $\dot{\phi} = \text{const.}.$

$$3M_{Pl}^2 H^2 = \rho, \quad M_{Pl}^2\left(3H^2 + 2\dot{H}\right) = -p$$

with $\rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X$

$$p = K - 2\left(G_\phi + G_X\ddot{\phi}\right)X$$

For $\rho = -p = -K = \text{const.} > 0$ we set $D \equiv K_X + 3gH\dot{\phi} = 0$

- The simplest solution

$$K(X) \equiv -X + \frac{X^2}{2M^3\mu} \quad \Rightarrow \quad X \simeq M^3\mu, \quad H^2 \simeq \frac{M^3\mu}{6M_{Pl}^2}.$$

$\mu = \text{const.}$

during de Sitter inflation

Generalization: Breaking the shift symmetry

★ Introduce a weak ϕ dependence on G to obtain a spectral tilt.

$$G(X) \equiv gX \Longrightarrow G(\phi, X) \equiv g(\phi) X$$

"slow-variation" parameters

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad \eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \varepsilon_g \equiv M_{Pl} \frac{g_\phi}{g} \ll 1 \Rightarrow$$

Quasi de Sitter inflation is an attractor.

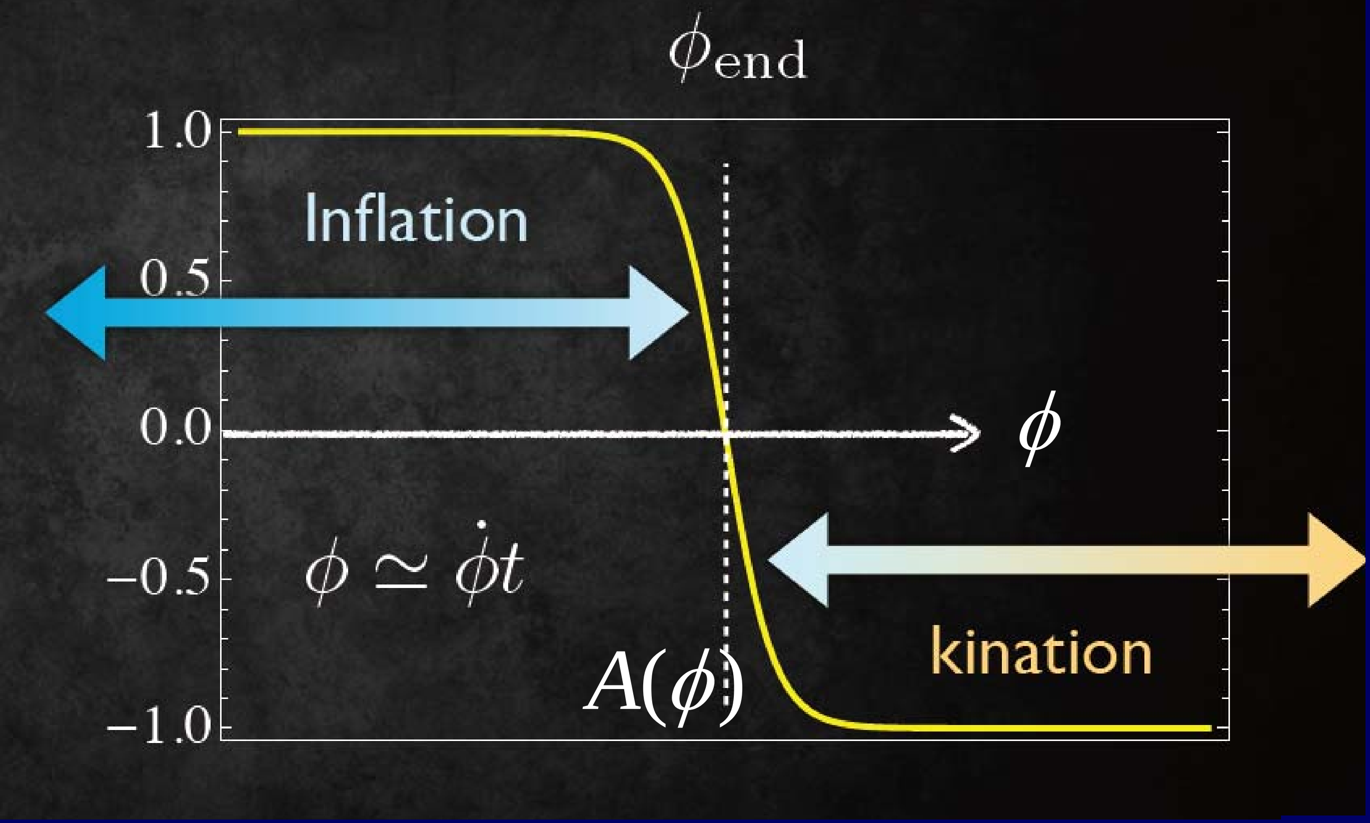
(NB $\dot{\phi}$ can be still large.)

★ $$K(X) \equiv -X + \frac{X^2}{2M^3\mu} \Longrightarrow K(\phi, X) \equiv -A(\phi) X + \frac{X^2}{2M^3\mu}$$

Inflation can be terminated by flipping the sign here.

A simple choice: $A(\phi) \equiv \tanh\left[\lambda\left(\phi_{end} - \phi\right)/M_{Pl}\right]$ with $\lambda = O(1)$.

Numerical solutions indicate ϕ stalls within one e-fold after crossing ϕ_{end} and all higher derivative terms become negligible. This function breaks the shift symmetry (severely) only in the vicinity of ϕ_{end}.



$K(X) \simeq X$

$$\rho = \frac{\dot{\phi}^2}{2} \propto a^{-6}(t).$$

$w = 1$

- ⋆ The Universe is reheated through gravitational particle production due to the change of the cosmic expansion law $a(t) \propto e^{H_{\rm inf} t} \to a(t) \propto t^{\frac{1}{3}}$. (Ford 87)
- ⋆ The Universe will eventually be dominated by radiation.

$$T_R \approx 0.01 \frac{H_{\rm inf}^2}{M_{Pl}} = 2 \times 10^7 \left(\frac{r}{0.1} \right) \mathrm{GeV}$$

r : tensor-to-scalar ratio

Curvature perturbations in G-inflation

We adopt the unitary gauge in which ϕ is homogeneous, $\delta\phi = 0$.

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2(1+2\mathcal{R})d\mathbf{x}^2$$

Curvature Perturbation

As usual,
① Expand the action to the second order.
② Eliminate α and β using constraint equations.
③ Obtain a quadratic action for $\mathcal{R}$.

$$\dot{\mathcal{R}} = \Theta\alpha,$$

$$\frac{\nabla^2}{a^2}\left(\mathcal{R} + a^2\Theta\beta\right) = X\mathcal{G}\alpha$$

$$S^{(2)} = \frac{1}{2}\int dt d^3x a^3\sigma\left[\frac{1}{c_s^2}\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\nabla\mathcal{R})^2\right],$$

$$\sigma = \frac{X\mathcal{F}}{M_{Pl}^2\Theta^2}, \quad \Theta = H - \frac{G_X\dot{\phi}^3}{2M_{Pl}^2}, \quad c_s^2 = \frac{\mathcal{F}}{\mathcal{G}},$$

where $\mathcal{F} = K_X + 2G_X(\ddot{\phi} + 2H\dot{\phi}) - 2\frac{G_X^2}{M_{Pl}^2}X^2 + 2G_{XX}X\ddot{\phi} - 2(G_\phi - XG_{\phi X}),$

$$\mathcal{G} = K_X + 2XK_{XX} + 6G_X H\dot{\phi} + 6\frac{G_X^2}{M_{Pl}^2}X^2 - 2(G_\phi - XG_{\phi X}) + 6G_{XX}HX\dot{\phi}.$$

Curvature perturbations in G-inflation

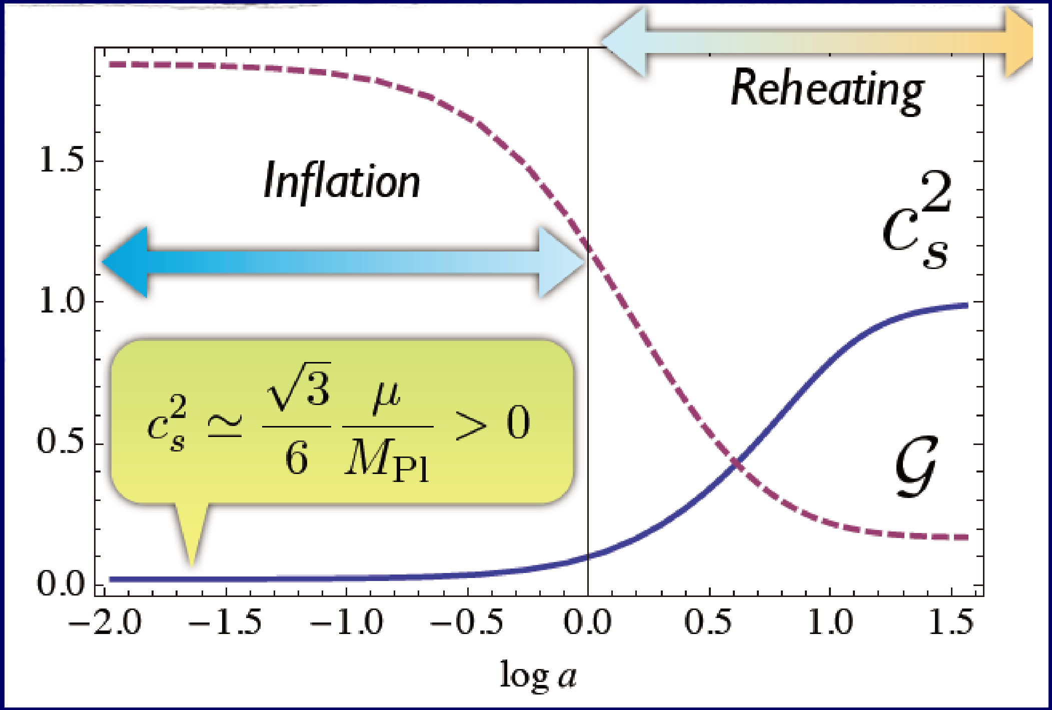
$$S^{(2)} = \frac{1}{2}\int dt d^3 x a^3 \sigma \left[\frac{1}{c_s^2}\dot{\mathcal{R}}^2 - \frac{1}{a^2}(\nabla \mathcal{R})^2 \right], \quad \frac{\sigma}{c_s^2} = \frac{X}{\Theta^2}\mathcal{G}.$$

No ghosts, No gradient instability if $\mathcal{G} > 0, \quad c_s^2 = \frac{\mathcal{F}}{\mathcal{G}} > 0.$

Our simple model satisfies these requirements.

$$K(\phi, X) \equiv -A(\phi)X + \frac{X^2}{2M^3\mu}$$

$$G(\phi, X) \equiv \frac{X}{M^3}$$



Power Spectrum $K(\phi, X), \quad G(\phi, X)$

New variables: $dy \equiv \frac{c_s dt}{a}, \quad \tilde{z} \equiv a\sqrt{\frac{2\sigma}{c_s}}, \quad u_k \equiv \tilde{z}\mathcal{R}$

Each Fourier mode satisfies

$$\frac{d^2 u_k}{dy^2} + \left(k^2 - \frac{\tilde{z}_{yy}}{\tilde{z}} \right) u_k = 0,$$

Variation parameters
(not necessarily small).

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \quad s \equiv \frac{\dot{c}_s}{Hc_s}, \quad \delta \equiv \frac{\dot{\sigma}}{H\sigma}.$$

We only assume they are nearly constant: $\frac{\dot{\varepsilon}}{H\varepsilon} \simeq 0, \quad \frac{\dot{s}}{Hs} \simeq 0, \quad \frac{\dot{\delta}}{H\delta} \simeq 0$

$$\frac{\tilde{z}_{yy}}{\tilde{z}} = \frac{q^2 - 1/4}{(-y)^2} \quad \text{with} \quad q \equiv \frac{3 - \varepsilon - 2s + \delta}{2(1 - \varepsilon - s)}.$$

The mode function reads $u_k = \frac{\sqrt{\pi}}{2}(-y)^{\frac{1}{2}} H_q^{(1)}(-ky), \quad q = \frac{3 - \varepsilon - 2s + \delta}{2(1 - \varepsilon - s)}.$

Power spectrum of curvature fluctuations

$$\mathcal{P}_{\mathcal{R}}(k)=\frac{4\pi k^3}{(2\pi)^3}\frac{|u_k|^2}{\tilde{z}^2}=2^{2q-3}\left|\frac{\Gamma(q)}{\Gamma(2/3)}\right|^2\frac{(1-\varepsilon-s)^2}{2\sigma c_s}\left(\frac{H}{2\pi M_{Pl}}\right)^2$$

@sound horizon crossing, $ky=-1$

Spectral index

$$n_s-1=3-2q=-\frac{2\varepsilon+s+\delta}{1-\varepsilon-s}$$

Variation parameters ε, s, & δ can be large, as long as this combination is small.

Curvature fluctuations are generated even in exact de Sitter background. ($\dot{\phi}\neq 0$). Power spectrum is scale-invariant then.

Tensor-to-scalar ratio

$$r\equiv\frac{\mathcal{P}_T(k)}{\mathcal{P}_{\mathcal{R}}(k)}=16\sigma c_s$$

can be as large as $r=0.17$, saturating the observational bound.

$$\sigma=-\frac{\dot{\Theta}}{\Theta^2}-\frac{\Theta-H}{\Theta},\quad\left(\Theta=H-\frac{G_X\dot{\phi}^3}{2M_{Pl}^2}\right)$$

$$\sigma\xrightarrow{G_X\to 0}\varepsilon$$

recovers the standard result.

Potential Driven G Inflation

Potential driven G-inflation

$$\mathcal{L}_\phi = X - V(\phi) - g(\phi) X \Box\phi,$$

Inflation is driven by the Potential.

Background equations of motion

$$3M_{Pl}^2 H^2 = \left[1 - gH\dot{\phi}(6-\alpha)\right] X + V(\phi)$$

$$M_{Pl}^2 \dot{H} = -\left[1 - gH\dot{\phi}(3+\eta-\alpha)\right] X$$

$$\left[3 - \eta - gH\dot{\phi}(9 - 3\varepsilon - 6\eta + 2\eta\alpha)\right] H\dot{\phi} + (1+2\beta) V'(\phi) = 0$$

Slow-roll parameters $\varepsilon = -\frac{\dot{H}}{H^2}, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \alpha = \frac{g'\dot{\phi}}{gH}, \quad \beta = \frac{g'' X^2}{V'(\phi)}.$

For $|\varepsilon|, |\mu|, |\alpha|, |\beta| \ll 1$ we find slow-roll EOMs

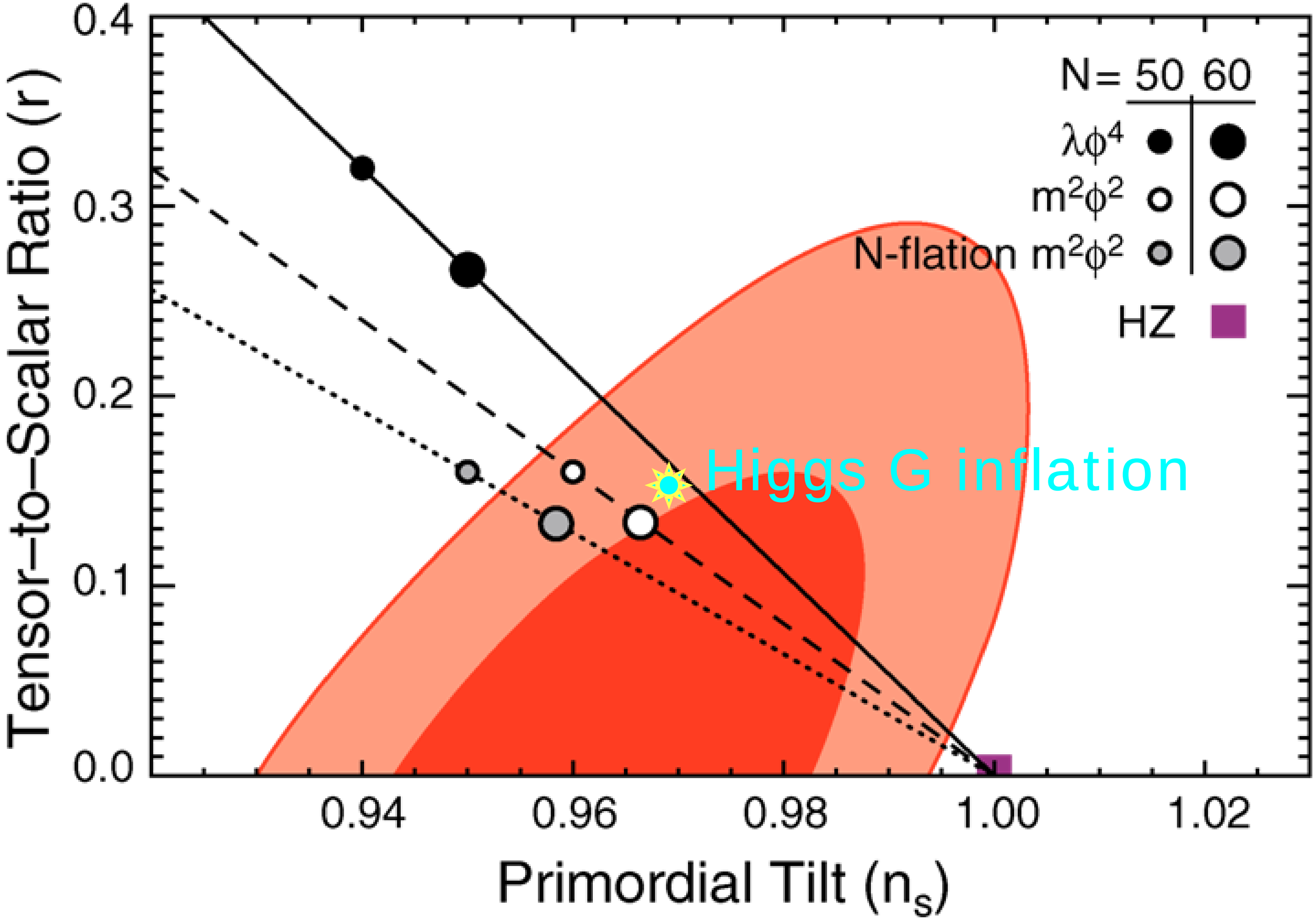
$$3H\dot{\phi}\left(1 - 3gH\dot{\phi}\right) + V'(\phi) \simeq 0 \qquad 3M_{Pl}^2 H^2 \simeq V(\phi)$$

This extra friction term enhances inflation and makes it possible to drive inflation by a standard Higgs field.

$$O_\phi = X - \frac{\lambda}{4}\phi^4 - \frac{X\phi\Box\phi}{M^4}$$

$$\mathcal{P}_{\mathcal{R}}(k) = 2.4\times10^{-9} \ @\ k = 0.002\mathrm{Mpc}^{-1} (\mathcal{N}_{\mathrm{COBE}} = 60)$$

$$M \simeq 4.7\times10^{-6}\lambda^{-\frac{1}{4}} M_{Pl} \simeq 10^{13}\,\mathrm{GeV}.$$



Nongaussianity

see also Mizuno and Koyama 1009.0677[hep-th]
Creminelli, D'Amico, Musso, Norena, Trincherini 1011.3004 [hep-th]
DeFelice and Tsujikawa 1103.1172[hep-th]
Noller and Magueijo 1102.0275[astro-ph.CO]

Nongaussianity of perturbations in G-inflation

We adopt the unitary gauge in which ϕ is homogeneous, $\delta\phi = 0$.

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\partial_i\beta dt dx^i + a^2(1+2\mathcal{R})d\mathbf{x}^2$$

Curvature Perturbation

As usual,
① Expand the action to the second order.
② Eliminate α and β using constraint equations.
③ Obtain a cubic action for $\mathcal{R}$.

$$S_3 = \int dt d^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot{\mathcal{R}}^3 + \mathcal{C}_2\mathcal{R}\dot{\mathcal{R}}^2 + \frac{\mathcal{C}_3}{a^4H^2}\partial^2\mathcal{R}(\partial\mathcal{R})^2 + \frac{\mathcal{C}_4}{a^2H^2}\dot{\mathcal{R}}^2\partial^2\mathcal{R} + \mathcal{C}_5 H\mathcal{R}^2\dot{\mathcal{R}} + \frac{\mathcal{C}_6}{a^4H}\partial^2\mathcal{R}(\partial\mathcal{R}\cdot\partial\chi) + \frac{\mathcal{C}_7}{a^4}\partial^2\mathcal{R}(\partial\chi)^2 + \frac{\mathcal{C}_8}{a^2}\mathcal{R}(\partial\mathcal{R})^2 + \frac{\mathcal{C}_9}{a^2}\dot{\mathcal{R}}(\partial\mathcal{R}\cdot\partial\chi) + \frac{2}{a^3}f(\mathcal{R})\left.\frac{\delta L}{\delta\mathcal{R}}\right|_1\right],$$

$$\chi := \partial^{-2}\Lambda, \quad \Lambda := \frac{a^2}{\Theta^2}X\mathcal{G}\dot{\mathcal{R}} = \frac{a^2\sigma}{c_s^2}\dot{\mathcal{R}}.$$

$$\mathcal{C}_1 = -\frac{H}{\Theta}\frac{\sigma}{c_s^2}\left(1+2\frac{\mathcal{I}}{\mathcal{G}}\right) - 2\dot{\phi}X\left(G_X + XG_{XX}\right)\frac{H\sigma}{c_s^2\Theta^2} + \frac{H^2\sigma}{c_s^4\Theta^2},$$

$$\mathcal{C}_2 = \frac{\sigma}{c_s^2}\left[3 - \frac{H^2}{c_s^2\Theta^2}\left(3+\epsilon+\frac{2\dot{\Theta}}{H\Theta}\right)\right],$$

$$\mathcal{C}_3 = -\frac{H^2\dot{\phi}XG_X}{\Theta^3},$$

$$\mathcal{C}_4 = \frac{2H^2\dot{\phi}X\left(G_X + XG_{XX}\right)}{\Theta^3},$$

$$M_{Pl} = 1$$

$$\mathcal{C}_5 = \frac{\sigma}{2c_s^2H}\frac{d}{dt}\left(\frac{H^2\delta}{c_s^2\Theta^2}\right),$$

$$\mathcal{C}_6 = \frac{2H\dot{\phi}XG_X}{\Theta^2},$$

$$\mathcal{C}_7 = \frac{\sigma}{4} - \frac{\dot{\phi}XG_X}{\Theta},$$

$$\mathcal{C}_8 = -\sigma + \frac{H^2}{\Theta^2}\frac{\sigma}{c_s^2}\left(1-\epsilon-2s-\frac{2\dot{\Theta}}{H\Theta}\right),$$

$$\mathcal{C}_9 = \frac{\sigma}{c_s^2}\left(-\frac{2H}{\Theta}+\frac{\sigma}{2}\right),$$

$$S_3 = \int dt d^3x\, a^3 \left[\frac{\mathcal{C}_1}{H} \dot{\mathcal{R}}^3 + \mathcal{C}_2 \mathcal{R} \dot{\mathcal{R}}^2 + \boxed{\frac{\mathcal{C}_3}{a^4 H^2} \partial^2 \mathcal{R} (\partial \mathcal{R})^2} + \boxed{\frac{\mathcal{C}_4}{a^2 H^2} \dot{\mathcal{R}}^2 \partial^2 \mathcal{R}} + \mathcal{C}_5 H \mathcal{R}^2 \dot{\mathcal{R}} \right.$$

$$\left. + \boxed{\frac{\mathcal{C}_6}{a^4 H} \partial^2 \mathcal{R} (\partial \mathcal{R} \cdot \partial \chi)} + \frac{\mathcal{C}_7}{a^4} \partial^2 \mathcal{R} (\partial \chi)^2 + \frac{\mathcal{C}_8}{a^2} \mathcal{R} (\partial \mathcal{R})^2 + \frac{\mathcal{C}_9}{a^2} \dot{\mathcal{R}} (\partial \mathcal{R} \cdot \partial \chi) + \frac{2}{a^3} f(\mathcal{R}) \left. \frac{\delta L}{\delta \mathcal{R}} \right|_1 \right],$$

$$\mathcal{C}_1 = -\frac{H}{\Theta} \frac{\sigma}{c_s^2} \left(1 + 2\frac{\mathcal{I}}{\mathcal{G}} \right) - 2\dot{\phi} X \left(G_X + X G_{XX} \right) \frac{H\sigma}{c_s^2 \Theta^2} + \frac{H^2 \sigma}{c_s^4 \Theta^2},$$

$$\mathcal{C}_2 = \frac{\sigma}{c_s^2} \left[3 - \frac{H^2}{c_s^2 \Theta^2} \left(3 + \epsilon + \frac{2\dot{\Theta}}{H\Theta} \right) \right],$$

$$\mathcal{C}_3 = -\frac{H^2 \dot{\phi} X G_X}{\Theta^3},$$

$$\mathcal{C}_4 = \frac{2H^2 \dot{\phi} X \left(G_X + X G_{XX} \right)}{\Theta^3},$$

$$\mathcal{C}_5 = \frac{\sigma}{2 c_s^2 H} \frac{d}{dt} \left(\frac{H^2 \delta}{c_s^2 \Theta^2} \right),$$

$$\mathcal{C}_6 = \frac{2H \dot{\phi} X G_X}{\Theta^2},$$

$$\mathcal{C}_7 = \frac{\sigma}{4} - \frac{\dot{\phi} X G_X}{\Theta},$$

$$\mathcal{C}_8 = -\sigma + \frac{H^2}{\Theta^2} \frac{\sigma}{c_s^2} \left(1 - \epsilon - 2s - \frac{2\dot{\Theta}}{H\Theta} \right),$$

$$\mathcal{C}_9 = \frac{\sigma}{c_s^2} \left(-\frac{2H}{\Theta} + \frac{\sigma}{2} \right),$$

$$\chi := \partial^{-2} \Lambda, \quad \Lambda := \frac{a^2}{\Theta^2} X \mathcal{G} \dot{\mathcal{R}} = \frac{a^2 \sigma}{c_s^2} \dot{\mathcal{R}}.$$

$$\mathcal{I} := X K_{XX} + \frac{2X^2}{3} K_{XXX} + H\dot{\phi} G_X + 6X^2 G_X^2 + 5H\dot{\phi} X G_{XX} + 6X^3 G_X G_{XX} + 2H\dot{\phi} X^2 G_{XXX} - \frac{2X}{3} \left(2G_{\phi X} + X G_{\phi XX} \right).$$

$$f(\mathcal{R}) = \frac{H\dot{\sigma}}{4c_s^2 \Theta^2 \sigma} \mathcal{R}^2 + \frac{H}{c_s^2 \Theta^2} \mathcal{R} \dot{\mathcal{R}} + \frac{1}{4a^2 \Theta^2} \left[-(\partial \mathcal{R})^2 + \partial^{-2} \partial^i \partial^j (\partial_i \mathcal{R} \partial_j \mathcal{R}) \right] + \frac{1}{2a^2 \Theta} \left[\partial \chi \cdot \partial \mathcal{R} - \partial^{-2} \partial^i \partial^j (\partial_i \mathcal{R} \partial_j \chi) \right]. \qquad \left. \frac{\delta L}{\delta \mathcal{R}} \right|_1 = a \left[\frac{d\Lambda}{dt} + H\Lambda - \sigma \partial^2 \mathcal{R} \right].$$

We calculate the bispectrum for non-slow-roll cases

assuming $\nu \equiv \dfrac{H - \Theta}{H} = const$ which means $\sigma = const$.

We also take $\dfrac{\mathcal{I}}{\mathcal{G}} = \mathcal{J}_1 + \dfrac{\mathcal{J}_2}{c_s^2}, \quad \dfrac{\dot\phi X^2 G_{XX}}{H} = \varrho_1 + \dfrac{\varrho_2}{c_s^2}.$

$$
\begin{aligned}
\mathcal{C}_1 &= \frac{\mathcal{D}_1}{c_s^2} + \frac{\mathcal{E}_1}{c_s^4}, \\
\mathcal{C}_2 &= \frac{\sigma}{c_s^2}\left[3 - \frac{1}{c_s^2}\frac{(1+\sigma)^2(3-\epsilon)}{(1+\epsilon)^2}\right], \\
\mathcal{C}_3 &= -\frac{(1+\sigma)^2(\sigma-\epsilon)}{(1+\epsilon)^3}, \\
\mathcal{C}_4 &= \mathcal{D}_4 + \frac{\mathcal{E}_4}{c_s^2}, \\
\mathcal{C}_6 &= \frac{2(1+\sigma)(\sigma-\epsilon)}{(1+\epsilon)^2}, \\
\mathcal{C}_7 &= \frac{4\epsilon - \sigma(3-\epsilon)}{4(1+\epsilon)}, \\
\mathcal{C}_8 &= -\sigma + \frac{1}{c_s^2}\frac{\sigma(1+\epsilon-2s)(1+\sigma)^2}{(1+\epsilon)^2}, \\
\mathcal{C}_9 &= \frac{\sigma}{c_s^2}\frac{-4+\sigma(\epsilon-3)}{2(1+\epsilon)}.
\end{aligned}
$$

Kinetically driven G inflation

$$
\mathcal{J}_1 = \text{const.}, \quad \mathcal{J}_2 = 0, \qquad \varrho_1 = \text{const.}, \quad \varrho_2 = 0.
$$

Potential driven G inflation

$$
\mathcal{J}_1 = \text{const.}, \quad \mathcal{J}_2 = 0, \qquad \varrho_1 = 0, \quad \varrho_2 = 0.
$$

$$
\begin{aligned}
\mathcal{D}_1 &= -\frac{\sigma(1+\sigma)}{1+\epsilon}\left[1 + 2\mathcal{J}_1 + 2\frac{\sigma-\epsilon+(1+\sigma)\varrho_1}{1+\epsilon}\right], \\
\mathcal{E}_1 &= -\frac{\sigma(1+\sigma)}{1+\epsilon}\left[2\mathcal{J}_2 - \frac{1+\sigma}{1+\epsilon}(1-2\varrho_2)\right], \\
\mathcal{D}_4 &= 2\frac{(1+\sigma)^3}{(1+\epsilon)^3}\left[\frac{\sigma-\epsilon}{1+\sigma} + \varrho_1\right], \\
\mathcal{E}_4 &= 2\frac{(1+\sigma)^3}{(1+\epsilon)^3}\varrho_2.
\end{aligned}
$$

We calculate the bispectrum using the in-in formalism

$$\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = -i\int_{t_0}^{t} \mathrm{d}t' \langle [\mathcal{R}(\mathbf{k}_1,t)\mathcal{R}(\mathbf{k}_2,t)\mathcal{R}(\mathbf{k}_3,t), H_{\rm int}(t')]\rangle,$$

$$H_{\rm int}(t) = -\int \mathrm{d}^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot{\mathcal{R}}^3 + \mathcal{C}_2\mathcal{R}\dot{\mathcal{R}}^2 + \cdots\right].$$

where t_0 is some early time when all modes were well within the horizon, while t is the time several e-folds after these modes left the Hubble radius.

$$\langle \mathcal{R}_{\mathbf{k}_1}\mathcal{R}_{\mathbf{k}_2}\mathcal{R}_{\mathbf{k}_3}\rangle = (2\pi)^7\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}_{\mathcal{R}}^2\frac{\mathcal{A}}{k_1^3k_2^3k_3^3}, \quad \mathcal{A} = \sum_M \mathcal{A}_M.$$

$$\begin{aligned}
\mathcal{A}_1 &= \frac{3}{2\sigma}(1-\epsilon-s)\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1}\left[\mathcal{D}_1 I_1(n_s-1)+\frac{\mathcal{E}_1}{c_{s*}^2}I_1(q')\right],\\
\mathcal{A}_2 &= \frac{1}{4}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1}\left[3I_2(n_s-1)-\frac{3-\epsilon}{c_{s*}^2}\left(\frac{1+\sigma}{1+\epsilon}\right)^2 I_2(q')\right],\\
\mathcal{A}_3 &= \frac{1}{2}\frac{\mathcal{C}_3}{\sigma c_{s*}^2}(1-\epsilon-s)^2\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_3(q'),\\
\mathcal{A}_4 &= \frac{3}{\sigma}(1-\epsilon-s)^2\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1}\left[\mathcal{D}_4 I_4(n_s-1)+\frac{\mathcal{E}_4}{c_{s*}^2}I_4(q')\right],\\
\mathcal{A}_6 &= \frac{\mathcal{C}_6}{8c_{s*}^2}(1-\epsilon-s)\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_6(q')\\
\mathcal{A}_7 &= \frac{\mathcal{C}_7}{4}\frac{\sigma}{c_{s*}^2}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_7(q'),\\
\mathcal{A}_8 &= \frac{1}{8}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1}\left[-I_8(n_s-1)+\frac{1+\epsilon-2s}{c_{s*}^2}\left(\frac{1+\sigma}{1+\epsilon}\right)^2 I_8(q')\right],\\
\mathcal{A}_9 &= \frac{\mathcal{C}_{9*}}{8}\left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2\left(\frac{k_1k_2k_3}{2k_t^3}\right)^{n_s-1} I_9(q'),
\end{aligned}$$

$$q' := \frac{s-2\epsilon}{1-\epsilon-s}.$$

$$\begin{aligned}
I_1(z) &:= \frac{k_1^2k_2^2k_3^2}{k_t^3}\cos\left(\frac{\pi z}{2}\right)\frac{\Gamma(3+z)}{2},\\
I_2(z) &:= \cos\left(\frac{\pi z}{2}\right)\left[\frac{2+z}{k_t}\sum_{i>j}k_i^2k_j^2-\frac{1+z}{k_t^2}\sum_{i\neq j}k_i^2k_j^3\right]\Gamma(1+z),\\
I_3(z) &:= \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\frac{2+z}{2}\left\{\Gamma(1+z)+\Gamma(2+z)\left[\frac{k_1k_2+k_2k_3+k_3k_1}{k_t^2}+(3+z)\frac{k_1k_2k_3}{k_t^3}\right]\right\}+\text{sym.},\\
I_4(z) &:= \frac{k_1^2k_2^2k_3^2}{k_t^3}\cos\left(\frac{\pi z}{2}\right)\frac{(6+z)\Gamma(3+z)}{12},\\
I_6(z) &:= \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\left[(3+z)\Gamma(1+z)+(3+z)\Gamma(2+z)\frac{k_3}{k_t}-\Gamma(3+z)\frac{k_3^2}{k_t^2}\right]+\text{sym.},\\
I_7(z) &:= \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\left[\Gamma(1+z)+\Gamma(2+z)\frac{k_3}{k_t}\right]+\text{sym.},\\
I_8(z) &:= \cos\left(\frac{\pi z}{2}\right)\left(\sum_i k_i^2\right)\left[\frac{k_t}{1-z}-\frac{1}{k_t}\sum_{i>j}k_ik_j-\frac{1+z}{k_t^2}k_1k_2k_3\right]\Gamma(1+z),\\
I_9(z) &:= \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)k_3^2}{k_t}\cos\left(\frac{\pi z}{2}\right)\left[(3+z)\Gamma(1+z)-\Gamma(2+z)\frac{k_3}{k_t}\right]+\text{sym.}.
\end{aligned}$$

The nonlinearity parameter

$$f_{NL} = 30 \frac{\mathcal{A}_{k_1=k_2=k_3}}{(k_1+k_2+k_3)^3}$$

$$\begin{aligned} f_{\rm NL} \;=\; & 30 \left|\frac{\Gamma(q)}{\Gamma(3/2)}\right|^2 \left(\frac{1}{54}\right)^{n_s-1} \left\{ \frac{3(1-\epsilon-s)}{2\sigma} \left[\mathcal{D}_1 I_1^{\rm equi}(n_s-1) + \frac{\mathcal{E}_1}{c_{s*}^2} I_1^{\rm equi}(q') \right] \right. \\ & + \frac{3(1-\epsilon-s)^2}{\sigma} \left[\mathcal{D}_4 \frac{n_s+5}{6} I_1^{\rm equi}(n_s-1) + \frac{\mathcal{E}_4}{c_{s*}^2} \frac{6+q'}{6} I_1^{\rm equi}(q') \right] \\ & + \frac{3}{4} I_2^{\rm equi}(n_s-1) + \frac{1}{4c_{s*}^2} \left[\frac{3\sigma^2}{8} - \frac{\sigma-\epsilon}{2(1+\epsilon)} - \frac{1+\sigma}{1+\epsilon} \left(\sigma + (3-\epsilon) \frac{1+\sigma}{1+\epsilon} \right) \right] I_2^{\rm equi}(q') \quad] . \\ & - \frac{(1-\epsilon-s)^2}{2\sigma c_{s*}^2} \frac{(1+\sigma)^2(\sigma-\epsilon)}{(1+\epsilon)^3} I_3^{\rm equi}(q') + \frac{3}{8(n_s-2)} I_6^{\rm equi}(n_s-1) \\ & \left. + \frac{1-\epsilon-s}{8c_{s*}^2} \frac{1+\sigma}{1+\epsilon} \left(3\frac{1+\sigma}{1+\epsilon} + 2\frac{\sigma-\epsilon}{1+\epsilon} \right) I_6^{\rm equi}(q') \right\} , \end{aligned}$$

$$I_M^{\rm equi}(z) := k_t^{-3} I_M(z)|_{k_1=k_2=k_3}$$

$$\begin{aligned} I_1^{\rm equi}(z) &= \frac{6}{z+6} I_4^{\rm equi}(z) = \cos\left(\frac{\pi z}{2}\right) \frac{\Gamma(3+z)}{1458}, \\ I_2^{\rm equi}(z) &= 2 I_7^{\rm equi}(z) = I_9^{\rm equi}(z) = \cos\left(\frac{\pi z}{2}\right) \frac{(4+z)\Gamma(1+z)}{81}, \\ I_3^{\rm equi}(z) &= \cos\left(\frac{\pi z}{2}\right) \frac{(2+z)(39+13z+z^2)\Gamma(1+z)}{2916}, \\ I_6^{\rm equi}(z) &= \frac{1-z}{3} I_8^{\rm equi}(z) = \cos\left(\frac{\pi z}{2}\right) \frac{(17+9z+z^2)\Gamma(1+z)}{243}, \end{aligned}$$

Generic result

$$f_{\rm NL} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{c_{\rm s}^2}\right) + \mathcal{O}\left(\tilde{\sigma}^2\frac{XG_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}}\right), \quad \tilde{\sigma} := \max\{1, \sigma\}.$$

$$\mathcal{I} := XK_{XX} + \frac{2X^2}{3}K_{XXX} + H\dot{\phi}G_X + 6X^2G_X^2 + 5H\dot{\phi}XG_{XX} + 6X^3G_XG_{XX} + 2H\dot{\phi}X^2G_{XXX} - \frac{2X}{3}\left(2G_{\phi X} + XG_{\phi XX}\right).$$

$$\mathcal{G} := K_X + 2XK_{XX} + 6G_XH\dot{\phi} + 6G_X^2X^2 - 2\left(G_\phi + XG_{\phi X}\right) + 6G_{XX}HX\dot{\phi}$$

$$\sigma = -\frac{\dot{\Theta}}{\Theta^2} - \frac{\Theta - H}{\Theta}, \quad \left(\Theta = H - \frac{G_X\dot{\phi}^3}{2M_{Pl}^2}\right) \quad \left(\sigma \xrightarrow{G_X \to 0} \varepsilon\right)$$

is not necessarily small.

Large $r = 16\sigma c_s$ and large f_{NL} are compatible.

★Potential driven G-inflation

NonGaussianity is small: $f_{NL} = \dfrac{235}{3888} \simeq 0.06.$

★ Kinetically driven G-inflation

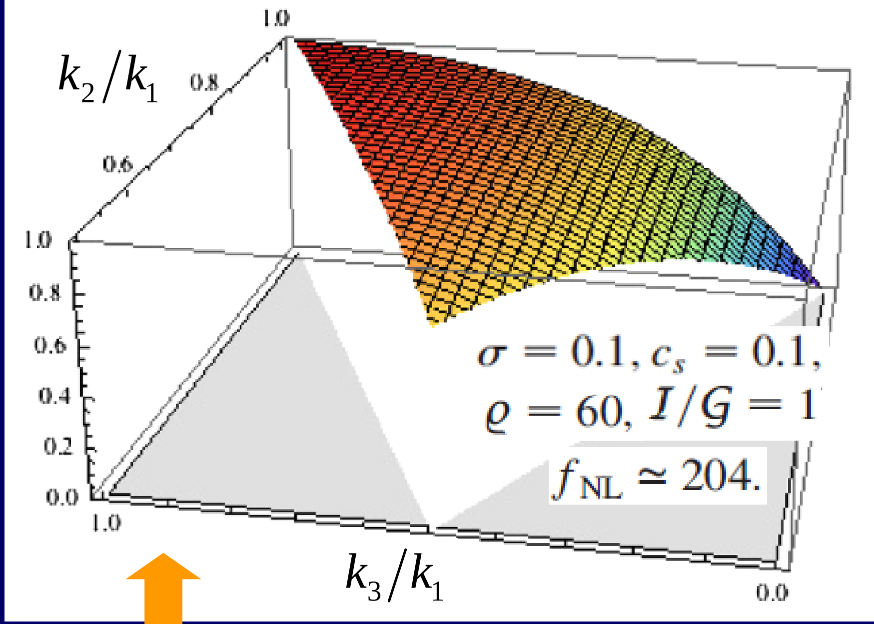
Nonlinearity parameter is determined by the four parameters $\sigma,\ c_{\rm s},\ ,\varrho \equiv \dfrac{\dot{\phi}X^2G_{XX}}{H},\ \dfrac{\mathcal{I}}{\mathcal{G}}.$

Contours of

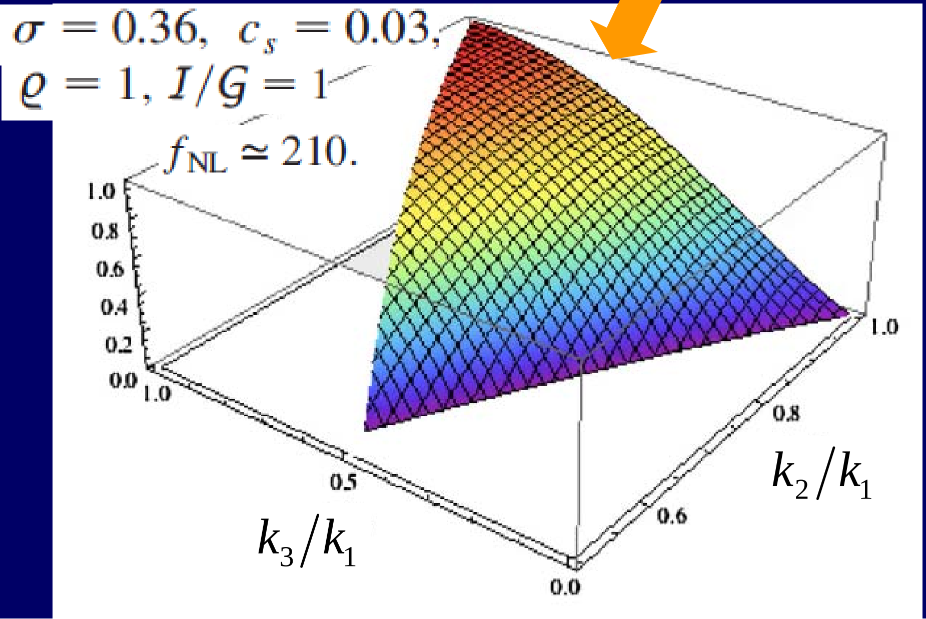
$\mathcal{A}(1, k_2/k_1, k_3/k_1)(k_2/k_1)^{-1}(k_3/k_1)^{-1}$

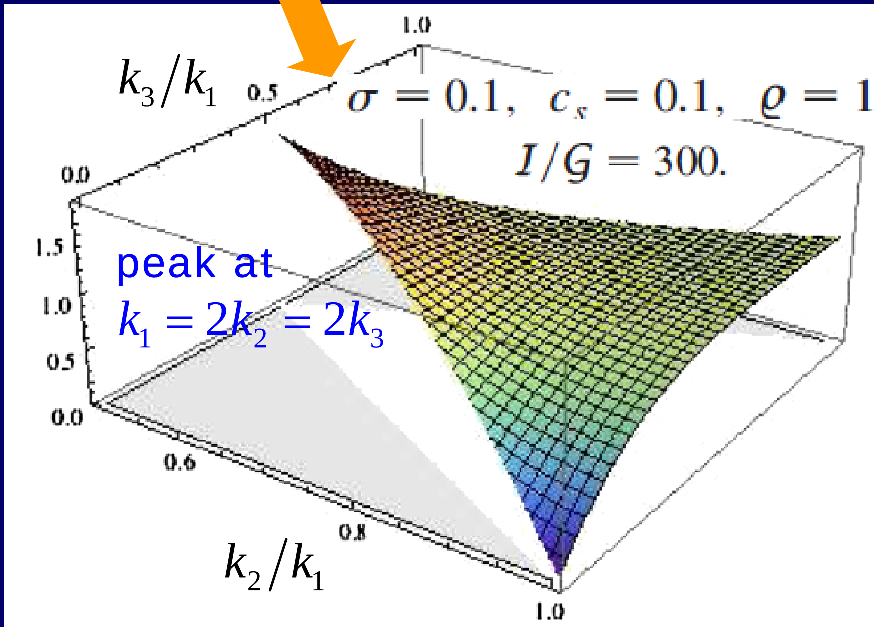
normalized to unity at the equilateral configuration for kinetically driven G inflation.

$$1 - k_2/k_1 \le k_3/k_1 \le k_2/k_1 \le 1$$



$$f_{\rm NL} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{c_s^2}\right) + \mathcal{O}\left(\tilde{\sigma}^2\frac{XG_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}}\right)$$





Conclusion

- G inflation is a new model of inflation containing a Galileon type derivative interaction $G(\phi, X)\Box\phi$.

- Kinetically driven G inflation can realize large tensor-to-scalar ratio and large nongaussianity at the same time.
  Curvature fluctuations can be generated even in the exact de Sitter background.

- Potential driven G inflation can yield a large tensor-to-scalar ratio but negligible nongaussianity.
  The standard Higgs field can be an inflaton.